International Journal of Physics
and Research (IJPR)
ISSN(P): 2250-0030; ISSN(E): 2319-4499
Vol. 4, Issue 1, Feb 2014, 37-44

STUDY OF THE COROLLARIES OF THE POINT SPREAD FUNCTION OF AN OPTICAL SYSTEM BY FOURIER ANALYTICAL METHODS USING PARABOLIC FILTERS

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ABSTRACT

In the absence of aberrations this diffraction-limited image which is assumed as a perfect system is the nearest approach to an ideal- image still it suffers from aberrations such as defocusing, spherical aberration, coma, astigmatism, *etc.*, in addition to the apodisation effects of diffraction.

This paper discusses some of the corollaries of the Point Spread Function (PSF) for the assessment of the image quality of an optical system namely; Strehl Ratio (SR) and Total Transmission Factor (τ) in order to improve the image quality of the system.

The results of the investigations carried out have been obtained with First- order parabolic filters by way of investigating the diffracted field and the imaging characteristics of an aberration- free rotationally symmetric optical system by employing Fourier Analytical Methods. The amplitude transmission curves for these filters have been be analytically expressed as $f(r) = (\alpha + \beta r^2)$ where β is the apodisation parameter which controls the non- uniform transmission of the pupil from point to point over its surface and ***r*** is the normalized distance of a point on the pupil from its centre and α numerical constant less than one and obviously appears as the D.C term or the average value of *f(r)*.

The Strehl Ratio curves for all values of α are straight lines parallel to each other i.e. having the same slope with respect to the apodization axis. The Total Transmission Factor increases with increasing values of α

KEYWORDS: Fourier Analytical Methods, Point Spread Function, First- Order Parabolic Filters, Strehl Ratio, and Total Transmission Factor

INTRODUCTION

The problem was probably first studied by Airy in 1834 and then developed by Lord Rayleigh by investigating the light distributions in the images of discs in the presence of spherical aberration. Rayleigh evaluated the central intensity by developing the diffraction integrals into series and indicated that the outer intensities might be calculated by the method of mechanical quadratures. The diffracted field characteristics of circular apertures with parabolic apodization filters will be studied at the Gaussian focal plane.

The ideal point spread function (PSF) is the three-dimensional diffraction pattern of light emitted from an infinitely small point source in the specimen and transmitted to the image plane through a high numerical aperture objective. It is considered to be the fundamental unit of an image in theoretical models of image formation. When light is emitted from such a point object, a fraction of it is collected by the objective and focused at a corresponding point in the image plane. However, the objective lens does not focus the emitted light to an infinitely small point in the image plane.

Rather, light waves converge and interfere at the focal point to produce a diffraction pattern of concentric rings of light surrounding a central, bright disk, when viewed in the x-y plane.

The mathematical theory was first established by LOMMEL [1], STRUVE [2], SCHARTZCHILD [3], and later by DEBYE [63]. Certain general features of the diffracted field near and far away from the Gaussian focal plane due to Airy type of pupils have already been established. In what follows the development of the mathematical formula for the complex amplitude of the diffracted light at a point in a specified image plane involves the method adopted by BORN and WOLF [4], the original treatment of which has been duly credited to DEBYE [5].

The distribution of the complex amplitude of the diffracted light using circular apertures with parabolic apodization filters has been obtained in the Gaussian focal plane. The diffracted field characteristics namely, Intensity Point Spread Function (Intensity PSF), the Strehl definition Ratio (SR) and the Total Transmission Factor(TTR) have been presented with the analysis of the results.

THE COMPLEX AMPLITUDE AND THE FIELD CHARACTERISTICS IN THE GAUSSIAN PLANE

In this section the expressions for the intensity at any point in the image plane of the PSF are obtained adopting parabolic apodization filters which are expressed mathematically as

$$f(r) = (\alpha + \beta r^2) \tag{1}$$

Where β is known as the apodization parameter and shows the degree of non-uniformity of transmission of the pupil. β =0 corresponds to Airy type of pupils. α is numerical constant less than one.

A circular aperture with the chosen filters with different values of the apodization parameter form a rotationally symmetric system is considered. The effect of these filters is only amplitude shading. Thus, an incident wave front is only affected in its amplitude, retaining its shape while passing through the system.

A spherical monochromatic wave has been considered to be emerging from an optical system and converging towards the axial focal point F as shown in the image formation scheme of Fig.1.

The diameter of the aperture be ‘2a’. The amplitude of the emergent wave at any point Q on the wave front is A f(r)/ R', where R' is the distance of F from the centre C of the wave front, S. This is also equal to the radius of the emerging wave front C, which momentarily fills the aperture.

A / R' is the amplitude of the incident wave at Q and f(r) defines the pupil function of the optical system under consideration

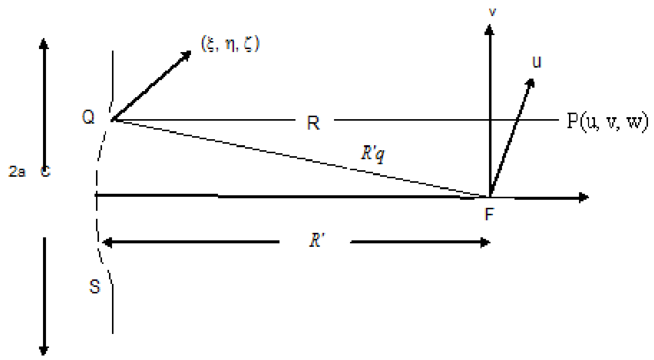

Figure 1: Diffraction at a Circular Aperture

A typical point P is considered in the neighborhood of F, specified by a vector $\vec{V}$ relative to F. The complex amplitude of the diffracted wave at P can be represented by G_A (p). It is assumed that the distance V=FP as well as the radius "a" (a>> λ) of the aperture are small compared to R'.

We get

$$G_A(p) = 2\pi i \sigma a^2 \int_0^1 f(r) \exp\left[\frac{-iyr^2}{2}\right] J_0(zr) r dr \qquad (2)$$

Where

$$\sigma = -\left(\frac{A}{\lambda R'^2}\right) \exp\left[i\left(\frac{R'}{a}\right)^2 y\right] \quad , \ldots \ldots\ldots\ldots \qquad (3)$$

These integrals were evaluated by LOMMEL [1] in terms of the well known Lommel functions introduced by LOMMEL himself. He expressed the complex amplitude in terms of convergent series of Bessel functions DURFEE [6] expressible again in terms of simple trigonometric functions and LOMMEL functions. He studied the intensity distributions in the geometrical focal plane. WOLF [7] has constructed the **isophotes** (the lines of equal intensity) near the focus from Lommel's data BORN and WOLF [5].

And the complex amplitude for a system of this type at the Gaussian focal plane y=0, is given by

$$G(0,z) = 2\int_0^1 \left(\alpha + \beta r^2\right) J_0(zr) r dr \ldots\ldots\ldots\ldots\ldots \qquad (4)$$

Where β is the apodization parameter; the pupil transmission is circularly symmetric and for $\alpha = 1$ and β =0, it reduces to Airy transmission.

$$G_A(0,z) = 2\int_0^1 J_0(zr) r dr \ \ldots\ldots\ldots\ldots\ldots \qquad (5)$$

Here, $G_A(0,z)$ is the diffracted field amplitude in the Gaussian focal plane. z is the distance of point in the image plane from the axis of the optical system, expressed in diffraction units. The intensity (*I*) at any point in the image plane of the PSF can easily be obtained from the following relation

$$I = GG^* \ \ldots\ldots\ldots \ldots\ldots \qquad (6)$$

Where G^* is the complex conjugate of G.

$$G_A(y,z) = 2\int_0^1 J_0(zr) \cos\left[\frac{yr^2}{2}\right] r dr - 2i\int_0^1 J_0(zr) \sin\left(\frac{yr^2}{2}\right) r dr \ \ldots\ldots\ldots\ldots\ldots \qquad (7)$$

INTENSITY PSF

The Point Spread Function (PSF) is the optical analogue of the Impulse Response Function (IRF) used in communication theory. Different researchers have defined the PSF in different ways though all of these definitions convey

the same meaning, only an ideal imaging system can reproduce an infinitesimally small point image .An ideal imaging system is one in which diffraction and aberrations are absent, further an imaging system is said to be perfect as the aberrations are absent. However, it is impossible to remove the effects of diffraction in the images formed by an optical system. That is why a perfect optical system is known as diffraction-limited system. The real optical imaging systems are, thus, neither ideal nor perfect, due to this, image of a point source object formed by any real optical system is never a point.

In figures 2 (a) to 2 (d) the computed values of IPSF for some typical values of α (0, 0.25, 0.50 & 0.75) and β (0, 0.3, 0.6 & 0.9) have been plotted. From the characteristics plotted the corollaries of the PSF namely, Strehl Ratio (SR) and the Total transmission Factor (TTF) have been discussed.

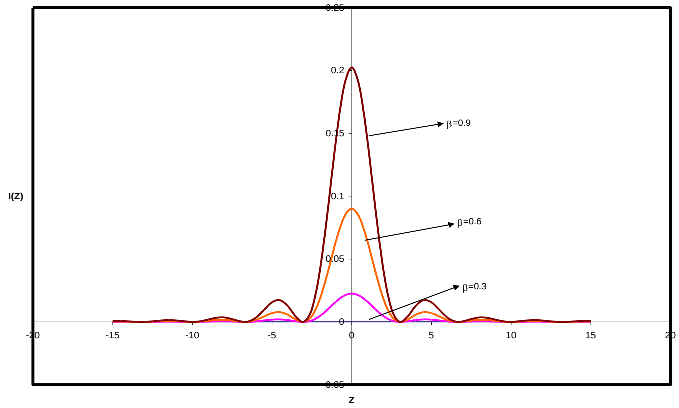

Figure 2(a): IPSF Curves for Various Values of β; α = 0

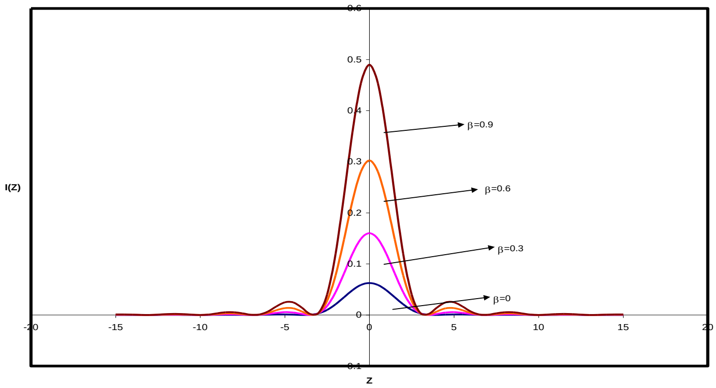

Figure 2(b): IPSF Curves for Various Values of β; α = 0.25

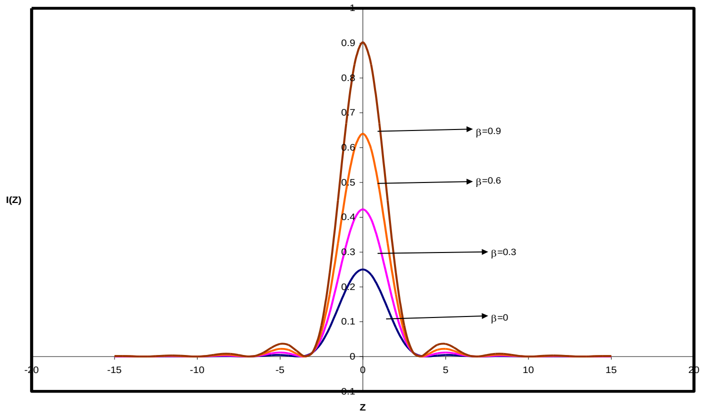

Figure 2(c): IPSF Curves for Various Values of β; α = 0.50

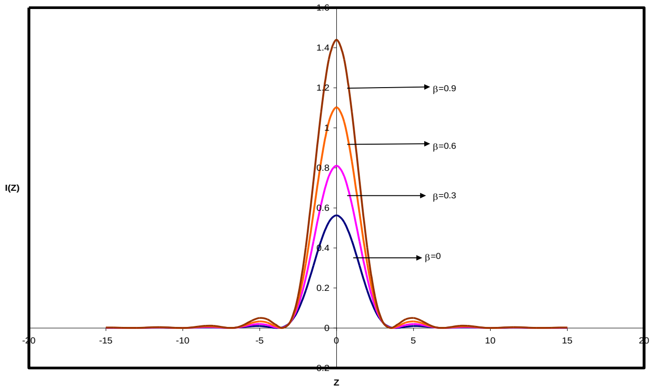

Figure 2(d): IPSF Curves for Various Values of β; α = 0.75

RESULTS AND DISCUSSIONS

It is evident from the figures 2 (a) to 2 (d), that for all permissible values of β for a particular value of α , the IPSF values decrease with decreasing values of β; however, the widths of the central maxima remain the same. In order to satisfy the **law of conservation of energy**, the decrease in the peak central energy is compensated by an equal amount of increasing energy in the first secondary maximum. And there is an upward shift of the point of congruence of the APSF curves with increasing values of α . The same phenomenon occurs in the case of IPSF curves also, though the shift of the point of congruence in the IPSF curves is much less compared to that in the APSF curves.

When the the location of first minimum, positions and the intensity values of the secondary maxima, for various values of β for a particular value of α are considered, it is interesting to observe that the locations of the first minimum and the secondary maxima are independent of the value of β and depend only on the values of α . As the value of α is increased the positions of first minima and the various secondary maxima increased, thereby reducing the resolving power of the system, according to the classical Rayleigh criterion resolution, however, and the value of α very high viz., α =0.75, there is a reversal of the above trend in the positions of the first minima and the secondary maxima.

COROLLARIES OF THE PSF

Strehl Ratio (SR)

STREHL [6] suggested the use of the **relative intensity** of the diffraction as a measure of the image quality. **The Strehl Ratio (SR)** is defined as the ratio of the central intensity of the PSF of the system and that of the uniform pupil function for diffraction limited system.

$$SR = \frac{I_p(0,0)}{I_A(0,0)} \quad \ldots\ldots\ldots\ldots\ldots \quad (8)$$

Where the subscripts P and A referred to the parabolic and Airy pupils respectively. According to KUSAKAWA [7], the above expression for SR can be written in terms of respective pupil function as follows.

$$SR = \frac{\left|G_p(0,0)\right|^2}{\left|G_A(0,0)\right|^2} \quad \ldots\ldots\ldots\ldots\ldots \quad (9)$$

$$SR = 4\left[\int_0^1 f(r)\,rdr\right]^2$$

$$SR = 4\left[\int_0^1 \left(\alpha + \beta r^2\right) rdr\right]^2 \quad \ldots\ldots\ldots\ldots\ldots \quad (10)$$

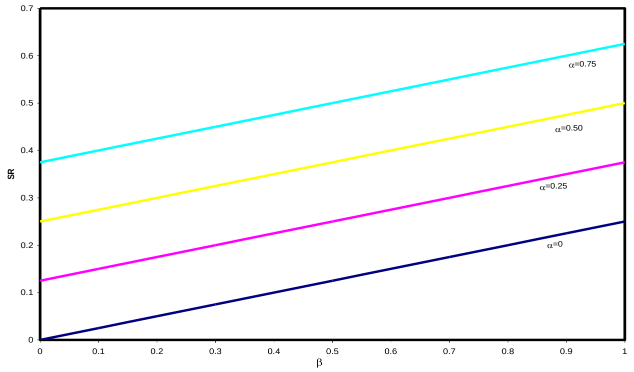

Figure 3: Variation of Sr with β; α = 0, 0.25, 0.50 & 0.75

In the figure 3, we have shown the variation of Strehl Ratio with β for values of $\alpha = 0$, 0.25, 0.50 & 0.75. It is observed from the figure that the SR- curves for all values of α are straight lines parallel to each other i.e. having the same slope with respect to the β - axis. These curves can, therefore be mathematically represented by the following straight line equation;

$$SR = m\beta + c \quad \ldots\ldots\ldots\ldots\ldots \quad (11)$$

Where m is the slope of the SR-curve and c is its intercept on the SR-axis. The important point, however, to be noticed is that the set of parabolic super-resolves we have chosen for our study doesn't yield an improved value of SR for the assessment of the image quality.

TOTAL TRANSMISSION FACTOR (τ)

The parameter ' τ ' i.e. the **Total Transmission Factor** is a very important parameter for non- Airy systems where apodization and central obscuration reduce the light flux. It is defined as.

$$\tau = \frac{\int_0^\infty \left|G_p(0,z)\right|^2 zdz}{\int_0^\infty \left|G_A(0,z)\right|^2 zdz} \quad \ldots\ldots\ldots\ldots\ldots \quad (12)$$

KUSAKAWA [7] has expressed it as the **Passing Flux Ratio** and has shown that, in terms of the pupil function, it can be written as

$$\tau = 2\int_0^1 \left|f(r)\right|^2 rdr \quad \ldots\ldots\ldots\ldots\ldots \quad (13)$$

For the parabolic system considered by us in the present study, equation (3.6) can be rewritten as

$$\tau = 2\int_0^1 \left|\left(\alpha+\beta r^2\right)\right|^2 rdr \quad \ldots\ldots\ldots\ldots\ldots \tag{14}$$

The total amount of light flux (F) received in the Gaussian focal plane due to non- Airy pupils can be expressed as:

$$F = \tau \int_0^\infty \left|G_A\left(0,z\right)\right|^2 zdz$$

or

$$F = 4\tau \int_0^\infty \left[\int_0^1 J_0\left(zr\right) rdr\right]^2 zdz \quad \ldots\ldots\ldots\ldots\ldots \tag{15}$$

The integral on the R.H.S of (3.8), on evaluation, gives the result.

$$F = 4\tau \int_0^\infty J_1\left(z\right) z^{-1} dz = 4\tau \left(\frac{1}{2}\right) = 2\tau \quad \ldots\ldots\ldots\ldots\ldots \tag{16}$$

The above result shows that for an Airy type of pupil τ =1 and it decreases in the presence of aberrations and apodization, for both circular and annular apertures.

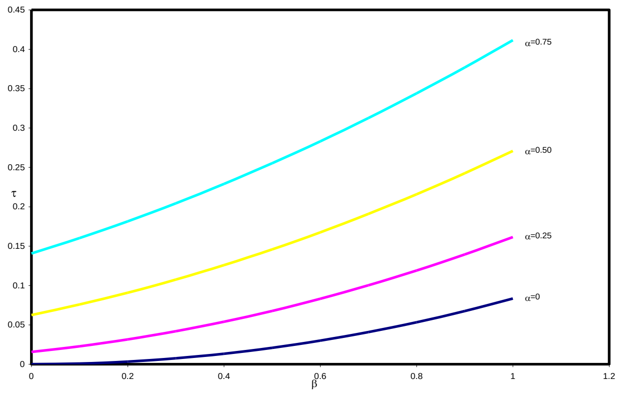

Figure 4: Variation of τ with β; α = 0, 0.25, 0.50 & 0.75

We have plotted the computed values of τ and shown it's variation with β for various values of α viz., $\alpha = 0, 0.25, 0.50 \,\&\, 0.75$ in the figure 4. It is observed from the figure that the total transmission factor (τ) increase with increasing values of α . This can be accounted for by analyzing the mathematical form of our chosen pupil function, i.e. $f\left(r\right)=\left(\alpha+\beta r^2\right)$. Here, α obviously appears as the D.C. term (or) the average value of $f\left(r\right)$

ACKNOWLEDGEMENTS

One of the authors Dr. Siva Prasad Peddi thanks his employer M/S Al Jouf University, Kingdom of Saudi Arabia, where he is working as a Professor, Department of Physics, College of Arts and Sciences, Al Qurayat, KSA.

This work has been presented in The IJAS Conference held at Freiburg in Germany between 03 and 06 Dec ’13 by one of the authors Dr. Siva Prasad Peddi

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